LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER - APRIL 2023

PST 3502 – STOCHASTIC PROCESSES

SECTION - A

3. When a state of a Markov chain is called (i) recurrent and (ii) positive recurrent?

4.State Abel lemma.

5. Show that communication is an equivalence relation.

6.Write the postulates of a birth and death process.

7.Explain the renewal process.

8.Define martingale and supermartingale of a Markov process.

9.Cite any two examples for branching process.

10. When a process is called (i) Stationary and (ii) Covariance stationary?

SECTION-B

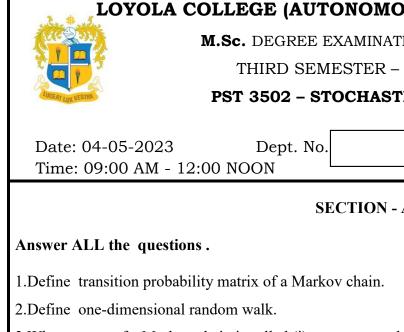
Answer any FIVE questions.

- 11. Prove that two-dimensional random walk is recurrent.
- 12. Let a Markov chain have four states 1,2,3 and 4 with the following one-step transition probabilities: $P_{12} = P_{13} = P_{14} = 1/3$, $P_{21} = P_{22} = P_{23} = 1/3$, $P_{32} = P_{34} = 1/2$ and $P_{41} = 1$. Find the stationary distribution.
- 13. State and prove the theorem used to find the stationary probability distribution when the Markov chain is positive recurrent, irreducible and aperiodic.
- 14. Derive mean and variance of the Yule process under the condition that X(0) = N = 1.
- 15. Explain Type II counter model in renewal process with the necessary diagram.
- 16. (a) Let $Y_0 = 0$ and Y_1, Y_2, \dots be independent and identically distributed random variables with mean 0 and variance σ^2 . If $X_0 = 0$ and $X_n = (Y_1 + Y_2 + ... + Y_n)^2 - n \sigma^2$ show that $\{X_n\}$ is a martingale with respect to $\{Y_n\}$. (4)
 - (b) Explain Wald's martingale. (4)
- 17. If m denotes the average number of offspring per individual and π the probability of extinction then show that $\pi = 1$ if $m \le 1$ and $\pi < 1$ if m > 1.
- 18. Explain (i) A stationary process on the circle and (ii) Stationary Markov chains. (4+4)

Max.: 100 Marks

 $10 \ge 2 = 20$ Marks

 $5 \times 8 = 40$ Marks



-2-SECTION –C

Answer any TWO questions.

2 x 20 = 40 Marks

- 19. (a) State and prove the basic limit theorem of Markov chains. (12)
 - (b) Derive the differential equations for a pure birth process by clearly stating the assumptions. (8)
- 20. Consider the state space $S = \{1,2,3,4,5,6\}$ with the one-step transition probabilities: $P_{11}=1/3$, $P_{13}=2/3$, $P_{22}=1/2$, $P_{23}=P_{25}=1/4$, $P_{31}=2/5$, $P_{33}=3/5$, $P_{42}=P_{43}=P_{44}=P_{46}=1/4$, $P_{55}=P_{56}=1/2$, $P_{65}=1/4$ and $P_{66}=3/4$.
 - (a) Draw the transition diagram and form the transition matrix. (2)
 - (b) Find the equivalence classes.(2)
 - (c) Determine period of states. (2)
 - (d) Show that states 1,3,5 and 6 are recurrent.(10)
 - (e) Prove that states 2 and 4 are transient.(4)
- 21. (a) Show that Poisson process can be viewed as a renewal process. (12)(b) Derive M(t) for a linear growth process with immigration. (8)
- 22. (a) Derive mean and variance for branching process. (12)
 - (b) State and prove the elementary renewal theorem. (8)

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