LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **APRIL 2023**

PST 3502 – STOCHASTIC PROCESSES

Date: 04-05-2023 Dept. No. Max. : 100 Marks

Time: 09:00 AM - 12:00 NOON

SECTION - A

Answer ALL the questions . **10 x 2 = 20 Marks**

1.Define transition probability matrix of a Markov chain.

2.Define one-dimensional random walk.

3.When a state of a Markov chain is called (i) recurrent and (ii) positive recurrent ?

4.State Abel lemma.

5.Show that communication is an equivalence relation.

6.Write the postulates of a birth and death process.

7.Explain the renewal process.

8.Define martingale and supermartingale of a Markov process.

9.Cite any two examples for branching process.

10. When a process is called (i) Stationary and (ii) Covariance stationary ?

SECTION –B

Answer any FIVE questions . **5 x 8 = 40 Marks**

11. Prove that two-dimensional random walk is recurrent.

12. Let a Markov chain have four states 1,2,3 and 4 with the following one-step transition probabilities: $P_{12} = P_{13} = P_{14} = 1/3$, $P_{21} = P_{22} = P_{23} = 1/3$, $P_{32} = P_{34} = 1/2$ and $P_{41} = 1$. Find the stationary distribution.

13. State and prove the theorem used to find the stationary probability distribution when the Markov chain is positive recurrent, irreducible and aperiodic.

14. Derive mean and variance of the Yule process under the condition that X(0) = N =1.

15. Explain Type II counter model in renewal process with the necessary diagram.

16. (a) Let $Y_0 = 0$ and $Y_1, Y_2, \ldots$ be independent and identically distributed random variables with mean 0 and variance σ^2. If $X_0 = 0$ and $X_n = (Y_1 + Y_2 + \ldots + Y_n)^2 - n\sigma^2$ show that $\{X_n\}$ is a martingale with respect to $\{Y_n\}$. (4)

(b) Explain Wald's martingale. (4)

17. If m denotes the average number of offspring per individual and π the probability of extinction then show that $\pi = 1$ if $m \leq 1$ and $\pi < 1$ if $m > 1$.

18. Explain (i) A stationary process on the circle and (ii) Stationary Markov chains. (4+4)

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SECTION –C

Answer any TWO questions . **2 x 20 = 40 Marks**

19. (a) State and prove the basic limit theorem of Markov chains. (12)
(b) Derive the differential equations for a pure birth process by clearly stating the assumptions. (8)
20. Consider the state space S = {1,2,3,4,5,6} with the one-step transition probabilities: P_{11}= 1/3 , P_{13} = 2/3 , P_{22} = 1/2, P_{23} = P_{25} = 1/4 , P_{31} = 2/5 , P_{33} = 3/5, P_{42} = P_{43} = P_{44} = P_{46} = 1/4, P_{55} = P_{56} = 1/2, P_{65} = 1/4 and P_{66} =3/4.
(a) Draw the transition diagram and form the transition matrix. (2)
(b) Find the equivalence classes.(2)
(c) Determine period of states. (2)
(d) Show that states 1,3,5 and 6 are recurrent.(10)
(e) Prove that states 2 and 4 are transient.(4)
21. (a) Show that Poisson process can be viewed as a renewal process. (12)
(b) Derive M(t) for a linear growth process with immigration. (8)
22. (a) Derive mean and variance for branching process. (12)
(b) State and prove the elementary renewal theorem. (8)

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